

# Some Features Of The Plane Couette Flow

Petr A. Skovorodko

*Institute of Thermophysics, Prospect Lavrentyev 1, 630090, Novosibirsk, Russia*

*E-mail: paskov@itp.nsc.ru*

**Abstract.** In the previous paper [1] it was found, in particular, that in the transition regime of the plane Couette flow the values of total energy flux and shear stress may exceed the corresponding free molecular values. For parameters of the problem (temperature ratio and speed ratio) considered in that paper, the discussed effect was sufficiently small (several percent). The present paper is devoted to more detailed investigation of this effect.

## INTRODUCTION

The plane Couette flow represents one of the simplest classical problem with the main features typical of a real flow with friction and heat transfer. Due to its one-dimensional character, the Couette flow is widely used for the purposes of testing the numerical algorithms [2].

In spite of many investigations of the plane Couette flow, some of its important features need to be studied more thoroughly. One of these features is the effect that in transition regime of the flow the values of total energy flux and shear stress may exceed the corresponding free molecular values [1]. As far as we know, this effect was not described previously.

For parameters of the problem (temperature ratio and speed ratio) considered in Ref. [1], the discussed effect is sufficiently small (several percent). In the present paper the more detailed investigation of this effect is made.

## PROBLEM FORMULATION

The steady one-dimensional flow of monatomic gas between two parallel plates is considered. One of the plates with the temperature  $T_1$  is fixed, while another with the temperature  $T_2$  is moving with the velocity  $U_2$ . For given molecular model the problem contains three determining parameters: the temperature ratio  $T_2 / T_1$ , the speed ratio  $S = U_2 / (2RT_1)^{1/2}$  and the Knudsen number  $Kn = \lambda / L$ ,  $\lambda$  being the mean free path and  $L$  – the distance between plates.

The problem is considered for solid spheres molecular model for the range of the Knudsen number  $0.1 \leq Kn \leq 100$ . The ratio  $T_2 / T_1$  was varied in the range  $1 \div 10$ , while the speed ratio  $S$  – in the range  $3 \div 30$ .

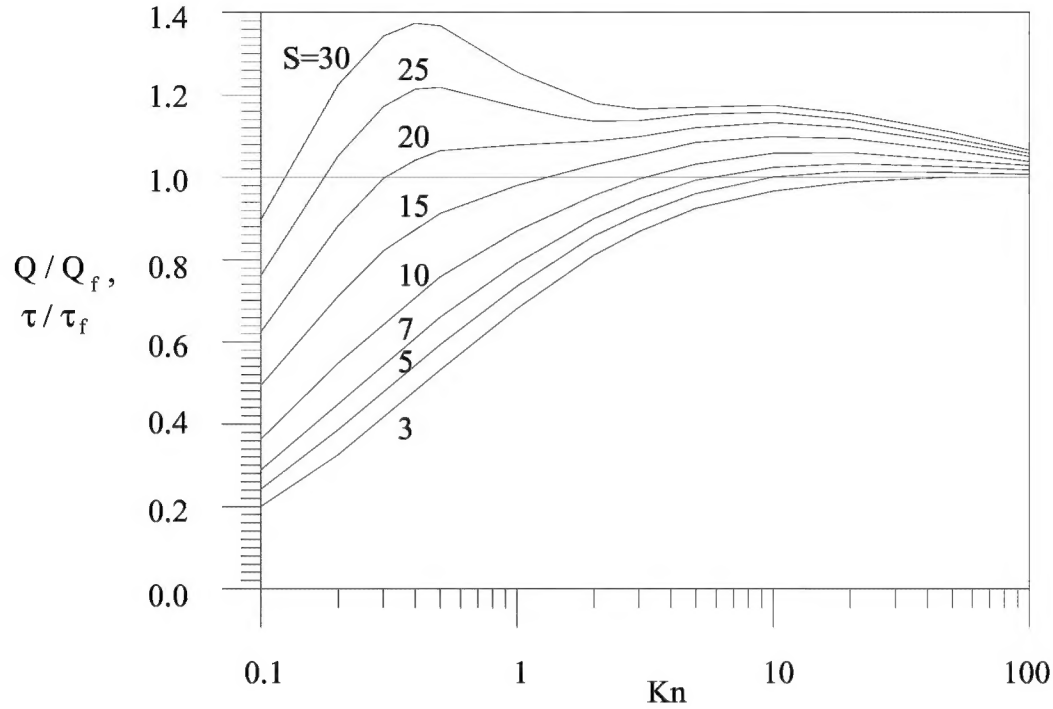
Standard no-time-counter DSMC method [2] was employed to simulate the motion and collision of molecules. The surfaces of plates are assumed to be diffusely reflected. The estimation of total energy flux  $Q$  and shear stress  $\tau$  was made using the sampling procedure that is based on the time that molecule stays in the cell. This procedure is shown to give the most accurate results of estimation [1].

Most of the results were obtained for uniform grid with  $N = 100$  cells with total number  $M$  of simulated molecules being equal to  $10^4$ . Some variants were recomputed with  $N = 200$  and  $M = 4 \cdot 10^4$ . As a result of test experiments the accuracy of the obtained results for  $Q$  and  $\tau$  is estimated to be better than 0.002. The results for  $Q$  and  $\tau$  are presented below in the form of ratio of these values to corresponding values  $Q_f$  and  $\tau_f$  for the free molecular flow.

| REPORT DOCUMENTATION PAGE  |                             |  |  | Form Approved OMB No.<br>0704-0188                       |  |
|--|-----------------------------|--|--|--|--|
| Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS. |                             |  |  |  |  |
| 1. REPORT DATE (DD-MM-YYYY)<br>09-07-2000  |                             | 2. REPORT TYPE<br>Conference Proceedings |  | 3. DATES COVERED (FROM - TO)<br>09-07-2000 to 14-07-2000 |  |
| 4. TITLE AND SUBTITLE<br>Some Features of the Plane Couette Flow<br>Unclassified   |                             |  |  | 5a. CONTRACT NUMBER                                      |  |
|  |                             |  |  | 5b. GRANT NUMBER   |  |
|  |                             |  |  | 5c. PROGRAM ELEMENT NUMBER                               |  |
| 6. AUTHOR(S)<br>Skovorodko, Petr A. ;  |                             |  |  | 5d. PROJECT NUMBER                                       |  |
|  |                             |  |  | 5e. TASK NUMBER  |  |
|  |                             |  |  | 5f. WORK UNIT NUMBER                                     |  |
| 7. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Institute of Thermophysics<br>Prospect Lavrentyev 1<br>630090 Novosibirsk, Russiaxxxxx  |                             |  |  | 8. PERFORMING ORGANIZATION REPORT<br>NUMBER              |  |
| 9. SPONSORING/MONITORING AGENCY NAME AND ADDRESS<br>AOARD<br>Unit 45002<br>APO AP, 96337-5002  |                             |  |  | 10. SPONSOR/MONITOR'S ACRONYM(S)                         |  |
|  |                             |  |  | 11. SPONSOR/MONITOR'S REPORT<br>NUMBER(S)                |  |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT<br>APUBLIC RELEASE   |                             |  |  |  |  |
| 13. SUPPLEMENTARY NOTES<br>See Also ADM001341, Rarefied Gas Dynamics (RGD) 22nd International Symposium held in Sydney, Australia, 9-14 July 2000.   |                             |  |  |  |  |
| 14. ABSTRACT<br>In the previous paper [1] it was found, in particular, that in the transition regime of the plane Couette flow the values of total energy flux and shear stress may exceed the corresponding free molecular values. For parameters of the problem (temperature ratio and speed ratio) considered in that paper, the discussed effect was sufficiently small (several percent). The present paper is devoted to more detailed investigation of this effect.   |                             |  |  |  |  |
| 15. SUBJECT TERMS  |                             |  |  |  |  |
| 16. SECURITY CLASSIFICATION OF:  |                             | 17. LIMITATION<br>OF ABSTRACT            |  | 18. NUMBER<br>OF PAGES                                   |  |
|  |                             | Public Release                           |  | 3  |  |
| 19. NAME OF RESPONSIBLE PERSON<br>Fenster, Lynn<br>lfenster@dtic.mil   |                             |  |  |  |  |
| a. REPORT<br>Unclassified  | b. ABSTRACT<br>Unclassified | c. THIS PAGE<br>Unclassified             | 19b. TELEPHONE NUMBER<br>International Area Code<br>Area Code Telephone Number<br>703767-9007<br>DSN<br>427-9007 |  |  |
|  |                             |  |  |  | Standard Form 298 (Rev. 8-98)<br>Prescribed by ANSI Std Z39.18 |

## RESULTS AND DISCUSSION

Fig. 1 illustrates the dependencies of  $Q / Q_f$  from the Knudsen number for the case  $T_2 / T_1 = 1$  for 8 values of speed ratio  $S$  in the range  $3 \div 30$ . It is known, that for the plates with equal temperatures the relation  $\tau / \tau_f = Q / Q_f$  takes place. In the simulations this relation was fulfilled with high accuracy.



**FIGURE 1.**  $Q / Q_f$  and  $\tau / \tau_f$  as a function of the Knudsen number for  $T_2 = T_1$ .

As can be seen from the presented data, for any Knudsen number the monotonous increase of  $Q / Q_f$  with increasing of  $S$  takes place. The studied effect ( $Q / Q_f > 1$ ) is observed for  $S > 5$ . For  $S = 30$  the maximum value of the effect is about 40% at  $Kn \approx 0.4$ . It is evident, that the effect will increase with further increasing of  $S$ .

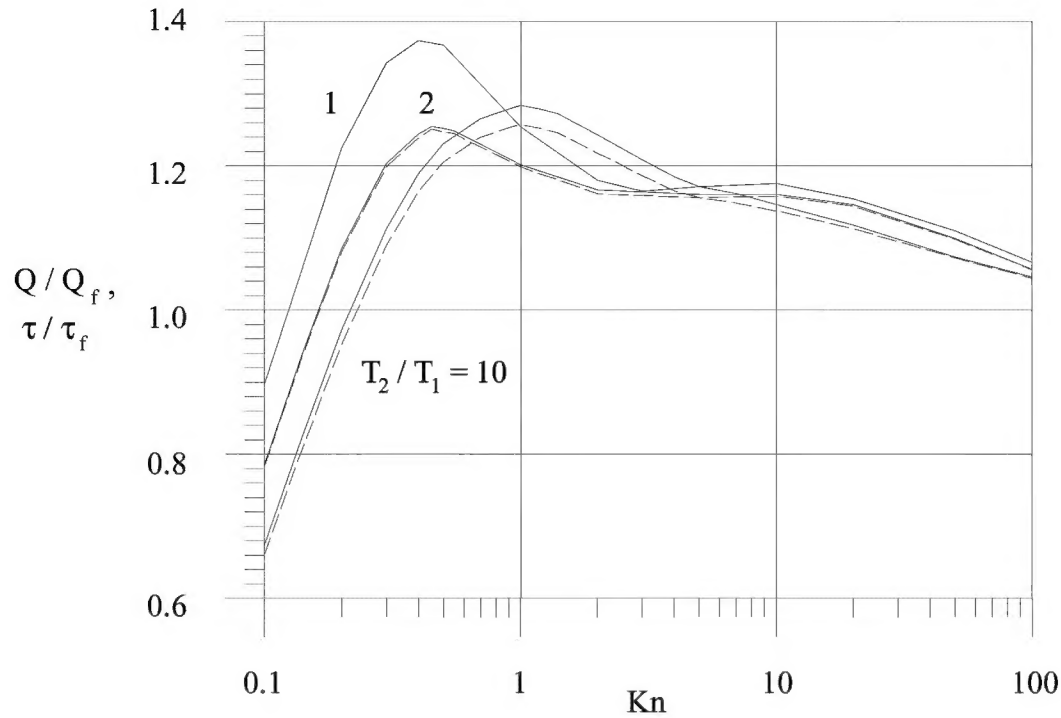
An interesting and unexpected feature of the presented dependencies consists in the changing of their shape at high values of  $S$ . Thus, at  $S > 25$  the dependence  $Q(Kn) / Q_f$  have two maxima – at  $Kn \approx 0.4$  and at  $Kn \approx 10$ . Further investigations are needed to explain these peculiarities.

The rearrangement of the flow caused by changing the ratio  $T_2 / T_1$  at constant  $S$  is illustrated in Fig. 2, where the dependencies of  $Q / Q_f$  (solid lines) and  $\tau / \tau_f$  (dashed lines) from  $Kn$  for  $S = 30$  and three values of  $T_2 / T_1$  (1, 2 and 10) are presented. The curve for  $T_2 / T_1 = 1$  is the same as presented in Fig. 1. For the case with  $T_1 \neq T_2$  the inequality  $\tau / \tau_f < Q / Q_f$  takes place, but the difference between these quantities is small (see Fig. 2).

As can be seen from Fig 2, at  $S = 30$  the increasing  $T_2 / T_1$  results in the transformation the curve with two maxima into the curve with one maximum at  $Kn \approx 1$ .

For heat transfer problem, that represents a special case of Couette flow with  $S = 0$ , the studied effect also take place at sufficiently high temperature ratio. Thus, at  $T_2 / T_1 = 100$  the heat flux  $Q$  1.028 times exceeds its free molecular value  $Q_f$  at  $Kn = 1$ .

The studied effect reflects the complex character of rearrangement of the distribution function from Navier-Stokes type at low Knudsen numbers to two half-Maxwellian one in the free molecular limit.



**FIGURE 2.**  $Q / Q_f$  and  $\tau / \tau_f$  as a function of the Knudsen number for  $S = 30$ .

## CONCLUSION

1. In the transition regime of the plane Couette flow the values of total energy flux and shear stress may significantly exceed the corresponding free molecular values due to the complex character of rearrangement of the distribution function from Navier-Stokes type at low Knudsen numbers to two half-Maxwellian one in the free molecular limit.
2. For the plates with equal temperatures the effect takes place for the range of speed ratio  $S > 5$ . For  $S = 30$  the maximum value of the effect is about 40% at  $Kn \approx 0.4$ . For high values of  $S$  ( $S > 25$ ) the dependence  $Q(Kn) / Q_f$  have two maxima – at  $Kn \approx 0.4$  and at  $Kn \approx 10$ .
3. For constant  $S$  the increasing  $T_2 / T_1$  may results in the transformation the curve  $Q(Kn) / Q_f$  with two maxima into the curve with one maximum at  $Kn \approx 1$ . For the case with  $T_1 \neq T_2$  the inequality  $\tau / \tau_f < Q / Q_f$  takes place, but the difference between these quantities is small.
4. For sufficiently high temperature ratio the effect is also observed for heat transfer problem ( $S = 0$ ).

## REFERENCES

1. Rebrov, A. K., Skovorodko, P. A., "An Improved Sampling Procedure in DSMC Method", in Rarefied Gas Dynamics-1996, edited by Ching Shen, 20th International Symposium Proceedings, Peking University Press, Beijing, China, 1997, pp. 215 - 220.
2. Bird, G. A., Molecular Gas Dynamics and the Direct Simulation of Gas Flows, Clarendon Press, Oxford, 1994.